May 2024

YBMA News

Vol. 33 No. 3

The Newsletter of the Yorkshire Branch of the Mathematical Association

If you eagerly awaited the AMiE merger prospectus promised for March 2024, you will have been disappointed. Plans to ask MA members to approve the formation of AMiE have been put on hold while discussions continue. A merger specialist has been brought in to guide the trustees and help to resolve current difficulties.

It remains unclear if and how the potential merger would affect branches. A recent survey of MA members found that the journals, the publications and the challenges were the most valued aspects of membership. If branch activities are not in this list, is it because most MA members are not aware of the events organised in their neighbourhood?

Membership of the MA is not a prerequisite for attending YBMA events. Our constitution expects us to encourage non-MA members to join the MA, but in almost 30 years I have not witnessed any strong-arm tactics! The YBMA stopped collecting an annual subscription 10 years ago and there are no plans to re-introduce it. The distinction between being a recognised YBMA member and an occasional attender at meetings has become blurred.

Our constitution proposes that we schedule at least one event per term. In pre-pandemic years we often managed to put on as many as seven events between October and June. Regular annual features have been our Christmas Quiz in early December and the W.P. Milne 6th Form Lecture shortly before Easter. Past MA presidents have usually given a talk to the YBMA in their year of office, a tradition that I hope will continue in future years.

I regret having been unable to fill the pages of recent newsletters with a plethora of forthcoming YBMA events. On the other hand, the section “Mathematics in the Classroom” has grown to compensate a little for this. You may feel the title is a misnomer and in some sense it is. The YBMA newsletter is for teachers and the challenges are aimed at them. The intention is never that they would become worksheets duplicated and handed out to a class. Yet I hope they always include ideas that suitably reworked or simplified can be used in a classroom setting.

Annual General Meeting 2024

Saturday, 8 June 2024
14:30 – 16:30

MALL 1, School of Mathematics
University of Leeds

Refreshments will be available at the end of the business part of the meeting.

The event will then conclude with some short presentations by YBMA members.

Alan Slomson

“Euclid”

Bill Bardelang

“DIY Langley-type Problems”

Further contributions most welcome.
Suggested time allowance 15 minutes.

A Date for your Diary

Wednesday, 4 December 2024
7pm for 7.30pm

YBMA Christmas Quiz

MALL 1, School of Mathematics
University of Leeds

YBMA Officers 2023-24

President: Lindsey Sharp (lindseyelizab50@hotmail.com)

Secretary & Newsletter: Bill Bardelang (rgb43@gmx.com)

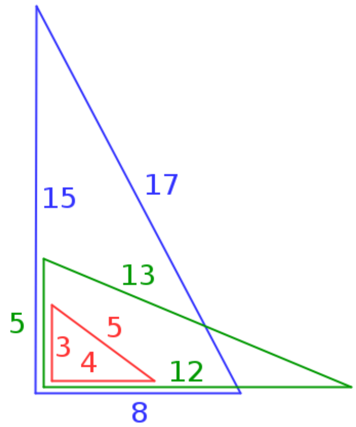
Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

Previous Newsletters can be found at

https://www.m-a.org.uk/branches/yorkshire

Mathematics in the Classroom

Integer Triangle Families



The 3, 4, 5 triangle has been used since ancient times to construct a right angle. It is far from unique in this respect and the diagram shows other integer triangles where the angle between the shortest sides is exactly 90°. We can say they belong to the Pythagorean family. In each case their side lengths a, b, c satisfy the equation

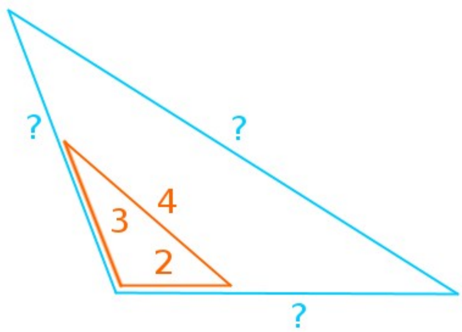
$$a^2 + b^2 = c^2$$

We can make further Pythagorean integer triangles using

$$a = l^2 - m^2, \quad b = 2lm, \quad c = l^2 + m^2$$

for any positive integers $l > m$ we choose.

Let us look at other consecutive integer triples. The 1, 2, 3 triangle has an angle of 180° between its shortest sides and other members of its family, i.e. sharing this angle property, will have sides l, m, $l + m$ for some positive integers l, m.

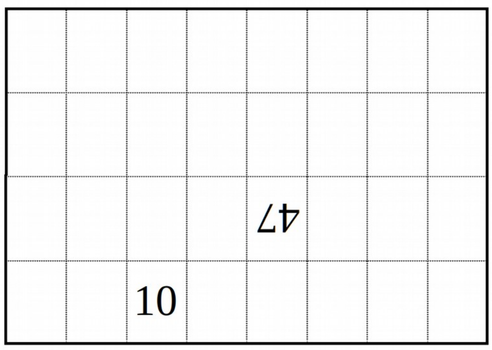


Somewhat more challenging is the triple 2, 3, 4. We invite readers to find the following:

(i) The equation satisfied by the sides a, b, c of a triangle so that the angle between its shortest sides is the same as for the 2, 3, 4 triangle.

(ii) A two-parameter solution of this equation.

Getting the order right – Solution



In the February Newsletter we asked readers to complete the page layout of an A0 sheet so that when folded, stapled and trimmed it becomes a 64-page A5 booklet. Every fold had to be either left half over right half or top half over bottom half. The position and orientation of pages 10 and 47 were to be as shown.

Five folds are needed and the creases will divide the A0 sheet into thirty-two A5-sized regions. The front and back of each region become two consecutive pages of the booklet. Let X denote the relation between front-and-back consecutive pages as just described. Focussing initially on relations and on page numbers only later, we treat our unfolded A0 sheet as 32 piles of 2 consecutive pages, X, X, X, . . . , X.

The first fold combines the piles pairwise, inverting one pile and placing it on top of the other. The top pages of the original piles will become consecutive pages at the centre of the newly formed pile. Let A be the relation between two consecutive pages thus created by the first fold. We then have 16 piles of 4 consecutive pages, XAX, XAX, . . . , XAX.

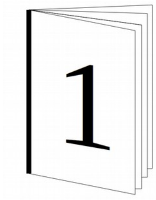
The second fold similarly creates 8 piles of 8 consecutive pages, XAXBXAX, XAXBXAX, . . . , XAXBXAX, and so on. After five folds we reach a single pile of 64 consecutive pages:

XAXBXAXCXAXBXAXDXAXBXAXCXAXBXAXEXAXBXAXCXAXBXAXDXAXBXAXCXAXBXAX

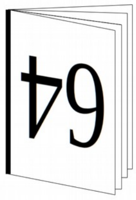
Given the location of any one particular page on the A0 sheet, this sequence will provide us with the means to find the location of subsequent or earlier pages. We can treat the relations as instructions:

X = move from front to back or vice versa,
A = move from the current position to its reflection in the crease produced by the 1st fold,
B = move from the current position to its reflection in the **nearest** crease produced by the 2nd fold, etc.

The restrictions imposed on the folding mean that the region in the bottom right hand corner of the A0 sheet stays where it is. The back of this region is never paired by folding, so it must be either page 1 or page 64. If our portrait-oriented A5 booklet is to open in the conventional way, i.e. so that pages are turned from right to left, it could then look like



or



Whenever we fold top half over bottom half, the pages in the top half are turned upside down. The bottom row pages are never turned upside down, so their orientation on the A0 sheet will match their orientation in the A5 booklet. Since page 10 has to be in the bottom row and the right way up, our booklet must have page 1 on top and 64 at the back. Shown below are the positions of pages 10 and 47 in our list of instructions.

XAXBXAXCX.AXBXAXDXAXBXAXCXAXBXAXEXAXBXAXCXAXBXA.XDXAXBXAXCXAXBXAX

1 ***10*** ***47*** ***64***

Instructions X, A, B, C, D, E are self-inverse and commute, i.e. applying any one of them twice has the effect of doing nothing and we can swap the order in which any two are applied without affecting the result. Starting from page 64, i.e. the back of the bottom right hand corner of the A0 sheet:

The 17 instructions to reach page 47 consist of 9X, 4A, 2B, 1C, 1D, which condense to XCD.

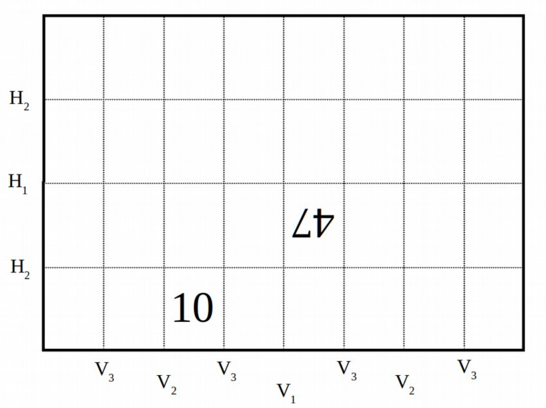
The 54 instructions to reach page 10 consist of 27X, 14A, 7B, 3C, 2D, 1E, which condense to XBCE.

We turn our attention to the creases. Creases V_1, V_2 and V_3 are formed by the first, second and third left-over-right folds respectively, similarly H_1 and H_2 by the first and second top-over-bottom folds.

The only order of folds which puts page 10 in the bottom row is

1st fold: A = top half over bottom half, producing crease H_1
2nd fold: B = left half over right half, producing crease V_1
3rd fold: C = left half over right half, producing crease V_2
4th fold: D = top half over bottom half, producing crease H_2
5th fold: E = left half over right half, producing crease V_3

The reader may want to check that this is also a satisfactory solution for page 47.



Working backwards from page 64 and using the full sequence of instructions, we can now fill in the whole of the A0 sheet, front and back.

59	six 6	11	54	51	14	3	62
38	27	22	43	46	19	30	35
39	26	23	42	47	18	31	34
58	7	10	55	50	15	2	63

As seen from the front

61	4	13	52	53	12	5	60
36	29	20	45	44	21	28	37
33	32	17	48	41	24	25	40
64	1	16	49	56	9 nine	8	57

As seen from the back